Agenda and list of abstracts

Imaging, Vision and Learning based on Optimization and PDEs

Bergen, Norway,
August 29 to September 1, 2016
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<td>8:00 am to 8:55 am</td>
<td>Reception</td>
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<tr>
<td>8:55 am to 9:00 am</td>
<td>Welcome</td>
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<tr>
<td>9:00 am to 9:45 am</td>
<td><strong>Morning session: Machine learning</strong></td>
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<tr>
<td>9:00 am to 9:45 am</td>
<td>Andrea Bertozzi</td>
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<tr>
<td>9:00 am to 9:45 am</td>
<td>Geometric graph-based methods for high dimensional data</td>
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<td>9:45 am to 10:30 am</td>
<td>Michael M. Bronstein</td>
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<td>Non-smooth manifold optimization with applications to machine learning and pattern recognition</td>
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<td>10:30 am to 11:00 am</td>
<td>Coffee break</td>
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<td>11:00 am to 11:45 am</td>
<td>Xavier Bresson</td>
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<td>11:00 am to 11:45 am</td>
<td>Convolutional Neural Networks on Graphs with Fast Localized Spectral Filtering</td>
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<td>11:45 am to 12:30 am</td>
<td>Kjersti Engan</td>
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<td>11:45 am to 12:30 am</td>
<td>Sparse representation and learned dictionaries in biomedical applications</td>
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<td>12:30 am to 2:00 pm</td>
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<td>2:00 pm to 2:45 pm</td>
<td><strong>Afternoon session: Processing of 3D data and manifolds</strong></td>
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<td>2:00 pm to 2:45 pm</td>
<td>Bob Plemmons</td>
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<td>Optimization Methods for 3D Image Data Recovery and Classification</td>
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<td>2:45 pm to 3:30 pm</td>
<td>Yang Wang</td>
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<td>2:45 pm to 3:30 pm</td>
<td>The Phase Retrieval Problem</td>
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<td>3:30 pm to 4:00 pm</td>
<td>Coffee break</td>
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<td>4:00 pm to 4:45 pm</td>
<td>Ronald Lok Ming Lui</td>
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<td>4:00 pm to 4:45 pm</td>
<td>TEMPO: Feature-endowed Teichmuller extremal mappings of point cloud for shape classification</td>
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<tr>
<td>4:45 pm to 5:30 pm</td>
<td>Alexander Vasiliev</td>
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<tr>
<td>4:45 pm to 5:30 pm</td>
<td>Evolution of smooth shapes: geometry and integrability</td>
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### Tuesday, August 30, 2016

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<th>Time</th>
<th>Session</th>
<th>Speaker</th>
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<tr>
<td>9:00 am to 9:45 am</td>
<td>Morning session: Numerical algorithms for PDEs and optimization problems</td>
<td>Joachim Weickert</td>
<td><em>FSI Schemes: Fast Semi-Iterative Solvers for PDEs and Optimisation Methods</em></td>
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<td>Michael Hintermueller</td>
<td><em>Bilevel Optimization and Applications in Imaging</em></td>
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<td>Yuesheng Xu</td>
<td><em>Solving Regularization Problems: Fixed-Point Proximity Algorithms vs ADMM</em></td>
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<td><em>Lunch break</em></td>
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<td>Afternoon session: Contributed posters</td>
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<td><em>Short presentations of each poster</em></td>
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<td><em>Poster session with refreshments</em></td>
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### Wednesday, August 31, 2016

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<td>9:00 am to 9:45 am</td>
<td>Morning session: Image restoration</td>
<td>Marcelo Bertalmío</td>
<td><em>Gamut extension for cinema</em></td>
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<td>9:45 am to 10:30 am</td>
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<td>Markus Grasmair</td>
<td><em>Local parameter adaptation in imaging applications by means of multi-resolution</em></td>
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<td><em>Coffee break</em></td>
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<td>11:00 am</td>
<td><strong>Final session: Convex variational models, relaxations and algorithms</strong></td>
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<td>Guy Gilboa</td>
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<td>Precise relaxation methods in image processing</td>
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<td>Image Recovery and Segmentation: Variational approach and Sparsity</td>
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<td>Convex variational methods on graphs - classification of high-dimensional data and segmentation of 3D point clouds</td>
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<td><em>Coffee break</em></td>
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<td>4:45 pm to</td>
<td><strong>Conclusions – outlook</strong></td>
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List of abstracts:

Invited presentations:

Andrea Bertozzi  
(UCLA, USA)

Title of contribution: Geometric graph-based methods for high dimensional data

Abstract of Contribution:  
We present new methods for segmentation of large datasets with graph based structure. The method combines ideas from classical nonlinear PDE-based image segmentation with fast and accessible linear algebra methods for computing information about the spectrum of the graph Laplacian. The goal of the algorithms is to solve semi-supervised and unsupervised graph cut optimization problems. I will present results for image processing applications such as image labeling and hyperspectral video segmentation, and results from machine learning and community detection in social networks, including modularity optimization posed as a graph total variation minimization problem.

Michael M. Bronstein  
(University of Lugano, Switzerland)

Title of contribution: Non-smooth manifold optimization with applications to machine learning and pattern recognition

Abstract of Contribution:  
Numerous problems in machine learning are formulated as optimization with manifold constraints, i.e., where the variables are restricted to a smooth submanifold of the search space. For example, optimization on the Grassman manifold comes up in multi-view clustering and matrix completion; Stiefel manifolds arise in eigenvalue-, assignment-, and Procrustes problems, compressed sensing, shape correspondence, manifold learning, sensor localization, structural biology, and structure from motion recovery; manifolds of fixed-rank matrices appear in maxcut problems and sparse principal component analysis; and oblique manifolds are encountered in problems such as joint diagonalization and blind source separation.

In this talk, I will present an ADMM-like method allowing to handle non-smooth manifold-constrained optimization. Our method is generic and not limited to a specific manifold, is very simple to implement, and does not require parameter tuning. I will show examples of applications from the domains of physics, computer graphics, and machine learning.
Abstract of Contribution:
Convolutional neural networks have greatly improved state-of-the-art performances in computer vision and speech analysis tasks, due to its high ability to extract multiple levels of representations of data. In this work, we are interested in generalizing convolutional neural networks from low-dimensional regular grids, where image, video and speech are represented, to high-dimensional irregular domains, such as social networks, telecommunication networks, or words' embedding. We present a formulation of convolutional neural networks on graphs in the context of spectral graph theory, which provides the necessary mathematical background and efficient numerical schemes to design fast localized convolutional filters on graphs. Numerical experiments on MNIST and 20NEWS demonstrate the ability of the system to learn local stationary features on graphs.

Title of Contribution: Convolutional Neural Networks on Graphs with Fast Localized Spectral Filtering

Kjersti Engan
(University of Stavanger, Norway)

Title of contribution: Sparse representation and learned dictionaries in biomedical applications

Abstract of Contribution:
The use of sparse representation and approximation together with learned, and often overcomplete, dictionaries has attracted considerable research interest in the last couple of decades. The sparse representation and learned dictionary approach has produced results comparable to state of the art in different applications, like image denoising, inpainting and texture classification among others.

Some approaches in dictionary learning will be presented, with an emphasis on the recursive least squares dictionary learning algorithm (RLD-DLA), which has been applied to different applications within biomedical signal and image processing in the literature. Some example applications within feature extraction and segmentation of cardiac magnetic resonance images, segmentation of the optic cup as well as inpainting of retina images will be discussed.

Predefined dictionaries or transforms, like Fourier transform or wavelets, inhabits a lot of structure in the atoms which can make such predefined dictionaries faster and easier to use. In general, a fixed and predefined dictionary can be useful for many types of images and signals and/or applications, whereas learned dictionaries typically are tailored for a specific signal or application. Combining these way of thinking, i.e. imposing structures in the atoms of a dictionary, but still learning a number of free variables to be specific for a certain application, defines new groups of interesting and useful dictionaries.

The flexible structured dictionary learning algorithm (FSD) is a new learning algorithm where different structures to the dictionary can be defined prior to learning. Shift invariant dictionaries are one example of such predefined structures. The FSD will be presented, as well as some results on representation of for example ECG signals using learned shift-invariant dictionaries.

Co-authors: Karl Skretting, Faraz Barzideh, Valery Naranjo,
Name: Bob Plemmons
(Wake Forest University, USA)

Title of contribution: Optimization Methods for 3D Image Data Recovery and Classification

Abstract of Contribution:
The goal of three dimensional (3D) imaging is to seek the complete structure of objects in our surroundings. Afterwards, we need to be able to extract information from the image data such as distance from the camera, as well as other information such as brightness and shape that are essential in a wide range of applications. Here we overview recent work with several co-authors for 3D image data recovery and classification, including (1) a summary of recent methods to enable snapshot 3D imaging with optimization-based image recovery, (2) trust-region algorithms for nonconvex sparse recovery of compressed sensing 3D hyperspectral data, and (3) classification of pixel-level fused hyperspectral and 3D LiDAR laser scanning data, using deep learning convolutional neural networks. Co-authors are J. Erway, R. Marcia, P. Pauca, S. Prasad, and T. Torgersen.

Name: Yang Wang
(HKUST, Hong Kong)

Title of contribution: The Phase Retrieval Problem

Abstract of Contribution:
In many applications such as X-ray Crystallography, imaging, communication and others one must construct a function/signal from only the magnitude of the measurements. These measurements can be, for example, the Fourier transform of the density function. While it is well known that we can recover a function from its Fourier transform, the classical phase retrieval problem asks whether we can recover a function from only the magnitude of its Fourier transform. The phase retrieval problem has since been extended to a much broader class of settings, referring to the reconstruction of a signal from only the magnitude of its linear measurements or more generally, from quadratic measurements. The problem, even in finite dimensions, turns out to be quite challenging. Many fundamental theoretical problems remain unresolved. Equally challenging is to develop fast and robust algorithms for phase retrieval. The problem has, not surprisingly, links to a many problems in science and engineering. But equally surprisingly it has also links to some classical problems on the embedding of projective spaces into Euclidean spaces and nonsingular bilinear forms. In this talk I'll give a brief overview and discuss some of the recent progresses.

Ronald Lok Ming Lui
(Chinese University of Hong Kong, Hong Kong)

Title of contribution: TEMPO: Feature-endowed Teichmuller extremal mappings of point cloud for shape classification

Abstract of Contribution:
In recent decades, the use of 3D point clouds has been widespread in computer industry. The development of techniques in analyzing point clouds is increasingly important. In particular, mapping of point clouds has been a challenging problem. In this talk, I will introduce a discrete analogue of the Teichmuller extremal mappings, which guarantees uniform conformality distortions, on point cloud
surfaces. Based on the discrete analogue, we develop a novel method called TEMPO for computing Teichmuller extremal mappings between feature-endowed point clouds. Using our proposed method, the Teichmuller metric can be computed for evaluating the dissimilarity of point clouds. Consequently, our algorithm enables accurate recognition and classification of point clouds. Experimental results will be demonstrated to show the effectiveness of our proposed method.

Alexander Vasiliev  
(Unciversity of Bergen, Norway)  
Title of contribution: Evolution of smooth shapes: geometry and integrability  
Abstract of Contribution:  
We consider smooth 2-D shapes evolving in time. Conformal reformulation in terms of welding allows us to describe the sub-Riemannian structure, which is natural for the space of shapes. On the other hand, we encode the evolution in the Sato-Segal-Wilson Grassmannian and relate it to a class of solutions to the KP hierarchy. Joint work with Irina Markina.

Joachim Weickert  
(Saarland University, Germany)  
Title of contribution: FSI Schemes: Fast Semi-Iterative Solvers for PDEs and Optimisation Methods  
Abstract of Contribution:  
Many tasks in image processing and computer vision are modelled by diffusion processes, variational formulations, or constrained optimisation problems. Basic iterative solvers such as explicit schemes, Richardson iterations, or projected gradient descent methods are simple to implement and well-suited for parallel computing. However, their efficiency suffers from severe step size restrictions. As a remedy we introduce a simple and highly efficient acceleration strategy, leading to so-called Fast Semi-Iterative (FSI) schemes. They extrapolate the basic solver iteration with the previous iterate. To derive suitable extrapolation parameters, we establish a recursion relation that connects box filtering with an explicit scheme for 1D homogeneous diffusion. FSI schemes avoid the main drawbacks of recent Fast Explicit Diffusion (FED) and Fast Jacobi techniques, and they have an interesting connection to the heavy ball method in optimisation.  
Our experiments show their benefits for anisotropic diffusion inpainting, nonsmooth regularisation, and Nesterov's worst case problems for convex and strongly convex optimisation.

Carola-Bibiane Schönlieb  
(University of Cambridge, UK)  
Title of contribution: Bilevel learning of variational inversion models  
Abstract of Contribution:  
TBC
Michael Hintermueller  
(Humboldt-University of Berlin, Germany)

Title of contribution: Bilevel Optimization and Applications in Imaging

Abstract of Contribution:
Recently, bilevel optimization has become a focus topic in mathematical image processing. It is of particular relevance in applications involving parameter learning.

This talk addresses opportunities as well as challenges connected to bilevel approaches in image restoration or blind deconvolution. While a major advantage is a monolithic optimization framework favoring leader-follower principles, some of the main challenges are related to non-smoothness, non-convexity, lack of constraint qualification as well as the design of efficient solution algorithms. Particular applications highlighted in this talk comprise distributed regularization parameter choice rules as well as topics in multiframe blind deconvolution.

Yuesheng Xu  
(Syracuse University, USA)

Title of contribution: Solving Regularization Problems: Fixed-Point Proximity Algorithms vs ADMM

Abstract of Contribution:
We shall discuss solving convex/non-convex regularization problems for image processing and machine learning. In particular, we shall compare the fixed-point proximity algorithms with ADMM.

Marcelo Bertalmio  
(University of Pompeu Fabra, Spain)

Title of contribution: Gamut extension for cinema

Abstract of Contribution:
Emerging display technologies are able to produce images with a much wider color gamut than those of conventional distribution gamuts for cinema and TV, creating an opportunity for the development of gamut extension algorithms that exploit the full color potential of these new systems.

In this talk we will present a novel GEA, implemented as a PDE-based optimization procedure related to visual perception models, that performs gamut extension by taking into account the analysis of distortions in hue, chroma and saturation. User studies performed using a digital cinema projector under cinematic (low ambient light, large screen) conditions show that the proposed algorithm outperforms the state of the art, producing gamut extended images that are perceptually more faithful to the wide-gamut ground truth, as well as free of color artifacts and hue shifts. We also show how none of the available image quality metrics is suitable for the gamut extension problem, as their results do not correlate with users' choices.
Markus Grasmair
(Norwegian University of Science and Technology, Norway)

Title of contribution: Local parameter adaptation in imaging applications by means of multi-resolution

Abstract of Contribution:
A well known challenge in variational approaches to image restoration, and also more general inverse problems, is that of parameter selection. Moreover, in order to obtain optimal reconstruction results, it is often necessary to apply local parameter selection strategies that take localised features of the image into account. A common approach to that goal bases the local regularisation parameters on statistical properties of the residual: The regularisation should be such that the residual is indistinguishable from independent Gaussian (or sub-Gaussian) noise.

In this talk we will take this approach and use it to develop a locally adaptive method that is based on the local behaviour of the residual on all possible scales present in the image. We will give a sound statistical justification for the method, and in addition prove that it is statistically asymptotically almost optimal in certain applications, for instance CT, provided that the assumptions on the noise, that is, independence and identical distribution, are correct.

The talk is based on a joint work with Housen Li and Axel Munk (University of Göttingen, Germany).

Guy Gilboa
(Technion, Israel)

Title of contribution: Spectral TV, Extensions and Connections to Sparsity

Abstract of Contribution:
Recent studies of convex functionals and their related eigenvalue problems show surprising analogies to harmonic analysis based on classical transforms, such as Fourier or wavelets. This enables new types of models and improved processing algorithms.

Recent formal results are presented for one-homogeneous functionals, such as total-variation, regarding an orthogonal decomposition based on nonlinear eigenfunctions. In addition, we discuss the relation of such decompositions to local and global sparsity constraints of general nonlinear smoothing evolutions.
Jan Lellmann  
(University of Lübeck, Germany)  

Title of contribution: Precise relaxation methods in image processing  

Abstract of Contribution:  
Energy-based method have become a popular tool for integrating prior knowledge in image and signal processing. Many interesting energies are nonconvex, which complicates optimization. However, in many applications it is possible to derive convex approximations which allow to find a good approximate minimizer. In this talk, we will give an overview of such techniques and show new results on how to greatly reduce the required problem size. Applications include image restoration, depth from stereo, and processing of cyclic and manifold-valued data.

Xiaoming Yuan  
(Hong Kong Baptist University, Hong Kong)  

Title of contribution: Some Recent Advances in Primal-Dual Methods for Saddle-Point Problems  

Abstract of Contribution:  
The primal-dual hybrid gradient (PDHG) method has found many applications in imaging because of its efficiency and simplicity in implementation. I will present an algorithmic framework of some generalized PDHG algorithms for saddle-point problems. This framework includes some popular algorithms as special cases; and particularly a completely symmetric PDHG algorithm with golden-ratio step sizes in both steps is developed based on this framework. Some inexact versions allowing for approximate solutions to PDHG’s subproblems are also developed. The worst-case convergence rates measured by iteration complexity and some asymptotically linear convergence rates of these new algorithms are also established.

Tieyong Zeng  
(Hong Kong Baptist University, Hong Kong)  

Title of contribution: Image Recovery and Segmentation: Variational approach and Sparsity  

Abstract of Contribution:  
We will address some convex/non-convex variational models for image recovery and segmentation. Most of them are related to non-Gaussian noise removal and sparsity driven issues. In particular, we shall present results for two-stage segmentation.

Egil Bae  
(Norwegian Defence Research Establishment, Norway)  

Title of contribution: Convex variational methods on graphs - classification of high-dimensional data and segmentation of 3D point clouds
Abstract of Contribution:

Variational problems on graphs have recently shown to be highly competitive for various data classification problems, but are inherently difficult to handle from an optimization perspective. This talk shows that a general subset of the classification problems can be closely approximated by a convex relaxation. Special cases include semi-supervised classification of high-dimensional data and a novel designed variational model for 3D point cloud segmentation. Multiple classes and certain penalty terms acting on the class sizes can be handled in the convex framework. In fact, both theoretical arguments and experimental evidence indicate that the approximation error tends to zero in practice. An efficient algorithm is presented based on a dual formulation of the convex relaxation. The algorithm is shown to be closely related to maximizing flow over the graph.
Poster presentations

Names: Pauline Tan and Antonin Chambolle

Title of contribution: Stereo Matching with Occlusion Detection in a Variational Framework

Abstract of Contribution:

This presentation is devoted to a new stereo matching algorithm which jointly estimates the disparity map and the occlusion sets. To proceed, we rely on a fine analysis of the occlusion phenomena, which is usually ignored in stereo methods, despite its consequences on the matching process.

We suppose that the images are rectified (fronto-parallel motion of the camera). Using an analysis found in [1], we showed that the disparity map satisfies the following properties when the monotonicity condition [2] holds:
1. where it is defined, the horizontal slope of the disparity map is strictly less than 1;
2. the width of each occlusion set is equal to the disparity gap before and after the occlusion.

Hence, if the disparity map is interpolated linearly on each line, then in the resulting map, the occlusions sets are characterized by a horizontal slope equal to 1. Moreover, this slope is less than 1 everywhere else. The idea is thus to define a functional which forbids this slope to exceed 1. The constraint is not violated when the matches are correct, and will be active when there is no possible matches, namely in occluded regions.

We thus consider the following functional
\[
E(u) = \int_{\Omega} g(x,u(x)) \, dx + TV(u) + \int_{\Omega} h^{vis}(\nabla u)
\]
where $h^{vis}(p^x) = \chi_{(-\infty;1]}(p^x_x)$ for each $p^x=(p^x_x,p^x_y)\in\mathbb{R}^2$. The quantity $g(x,u(x))$ measures the dissimilarity between $x$ in the reference image and $x-u(x)$ in the other view. It defines the data term. The map $g$ is supposed to be continuous on $\Omega\times\mathbb{R}$. The second term is a regularization term, which is here based on the TV semi-norm. It has been showed in [3] that, despite its nonconvexity, this kind of functional leads to a convex problem, thanks to a change of variables and a convex relaxation. By using a proper set of dual variables, the resulting relaxed problem can then be solved by a primal dual algorithm. We also proposed an accelerated algorithm which solves a equivalent but strongly convex problem.

The talk is based on joint work with Antonin Chambolle.

References:
Abstract of Contribution:
Game theoretic stochastic or deterministic methods have recently emerged as a novel approach to study and to approximate various non-linear partial differential equations (PDE). In particular Tug-of-War Games (TOG) related to the $\infty$-Laplacian or to the $p$-Laplacian have attracted a lot of attention. They were first introduced by Peres, Schramm, Sheffield, and Wilson in [1, 2].

It is now used in many works to study the existence or the regularity of solutions for many PDEs. Many of these games generally are formulated as well-known statistical functionals such as mean, min, or max operators. They are interpreted as a discrete approximation of the underlying PDE and solving the latter leads to taking a suitable limit of the solution of the discrete game.

Recently, there is a high interest in adapting and solving PDEs on data which is given by arbitrary graphs and networks. The demand for such methods is motivated by existing and potential future applications, such as in machine learning and mathematical image processing. Indeed, any kind of data can be represented by a graph in an abstract form in an abstract form in which the vertices are associated to the data and the edges correspond to relationships within data.

In this paper, we consider several Dynamic Programming Equations arising in the discrete game-theoretic interpretation for various non-linear PDEs including $\infty$-Laplacian, game $p$-Laplacian and Hamilton-Jacobi related equations. We show that under our framework of Partial difference equations these discrete games coincide with PDEs on particular Euclidean graphs.

The same PDEs on weighted graph of arbitrary topology lead to non-local statistical functionals, namely non-local means, non-local dilation, and non-local erosion operators. We interpret them as non-local Tug-of-War Games and we show their connections to non-local PDEs on Euclidean graphs.

This paper unfolds as follows: first, we briefly introduce several discrete games and their related PDE. Next, we introduce our previously proposed PdE framework, and show how the games can be viewed under this framework. Then, we naturally extend these games to non-local forms, and conclude by presenting some experiments on image and point clouds processing.

Names: Michael Hintermüller, Kostas Papafitsoros and Carlos Rautenberg

Title of contribution: A fine scale analysis of spatially adapted total variation regularisation

Abstract of Contribution:

Spatially adapted regularisation in image reconstruction has been used in order to preserve details like texture and other highly oscillating features that are naturally present in the image. The idea is to apply regularisation of different strength in different parts of the image by spatially varying the regularisation parameters. In this work, we study analytically the effect of the spatially varying parameters to the structure of solutions of total variation type regularisation.

Name: Nafaa Nacereddine, Aicha Baya Goumeidane and Djemel Ziou

Title of contribution: Unsupervised classification of weld defects in radiographic images based on multivariate generalized Gaussian mixture model

Abstract of Contribution:

Mainly used in the petroleum, petrochemical, nuclear and power generation industries especially, for the inspection of welds, the radiography has played an important role in the quality assurance of the piece or component, in conformity with the requirements of the standards, specifications and codes of manufacturing. The radiographic testing of welds is a nondestructive method that uses the penetrating and ionizing inspection radiation to detect internal discontinuities such as, porosity, crack, lack of penetration, solid inclusion, etc. The radiography expert has as role to inspect manually each film in order to detect the presence of possible defects which he must then identify and measure. The expert often works in the extreme cases of the visual system because of many factors (low dimension of some defects, bad contrast, image noise, etc.) making the subjectivity, in the mechanisms of weld defect detection and classification, not negligible.
Thus, the computer aided analysis systems, including image processing, pattern recognition and statistics are developed in the radiographic image interpretation, in order to improve the accuracy and the objectivity of evaluation and also to make the inspection process automatic, faster and more reliable. The computer vision dedicated to radiographic image inspection consists generally in five stages: film digitization, preprocessing, image segmentation, feature extraction and classification.

In the present work, while the segmentation and the feature extraction stages are performed using region-based active contours and the geometric measures-based shape descriptors, respectively, the novelty in this paper deals with the classification task which uses the mixture model of multivariate generalized Gaussian distribution (MGGD) in such an application. The Expectation-Maximization algorithm is used to estimate the parameters of the GGD mixture which fits the M-class weld defect data represented in d-dimensional feature space corresponding to the d geometric measures.

An accurate modeling of unknown probability density functions pdfs of data, encountered in practical applications, can play an important role in machine learning, clustering and pattern recognition. Including Gaussian, Laplacian and uniform distributions as special cases, MGGD is potentially interesting for modeling the statistical properties of computer vision applications. In fact, the GGD is an elliptically contoured distribution characterized not only by a mean vector and a scatter matrix, but also by a shape parameter determining the peakiness of the distribution and the heaviness of its tails making this distribution more flexible than multivariate Gaussian distribution (MGD) and thus, more suitable for modeling, among others, images or features extracted from these images.

However, the expressions of the partial derivative equations (PDE) deriving the MGGD parameters handle highly nonlinear functions including piece-wise, logarithm, gamma, psi, power, etc. So, a particular attention is required for the derivatives computation, especially, for the matrix differentiation. Here, the solutions are given by the Newton-Raphson method.

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Names: Atsushi Imiya, Hayato Itoh and Tomoya Sakai

Title of contribution: Relaxed Optimisation for Tensor Principle Component Analysis and Applications to Recognition, Compression and Retrieval of Volumetric Shapes

Abstract of Contribution:

A pattern is assumed to be a square integrable function in a linear space and to be defined on a finite support in n-dimensional Euclidean space. For planar and volumetric pattern, the dimensions of Euclidean spaces are two and three, respectively. For numerical computation in pattern recognition, we deal with sampled patterns. In traditional pattern recognition, these sampled patterns are embedded in an appropriate-dimensional Euclidean space as vectors. The other way is to deal with sampled patterns as multi-way array data. These multi-way array data are expressed as tensors to preserve multi-linearity of data in the original pattern space, since tensors allow to express multi-way array data in multi-linear forms.

For application of the multi-way principal component analysis (PCA) to volumetric data analysis, we develop a relaxed closed form for orthogonal decomposition of tensors based on the Tucker-3 method. Although the Tucker-3 tensor decomposition is achieved by solving variational optimisation iteratively, our method solves a system of variational optimisation problems derived from the original expression of the Tucker-3 decomposition with the orthogonal constraints for solutions. Furthermore, we numerically clarify that the discrete cosine transform (DCT) matrix efficiently approximates the solution of the original problem, since the cosine transform of Markov-1 signals approximates Karhunen-Loeve (KL) transform.

The method allows to compress and retrieve volumetric data preserving both geometric and statistic properties as objects and textures, respectively. Moreover, this orthogonal-projection-based data compression method for multi-way data array allows to extract outline shapes of biomedical objects such as organs and interior structures of cells.
Organs, cells in the organ and micro-structure in cells, which are dealt with in biomedical image analysis, are spatial texture. Furthermore, these biological objects possess spatial geometric and topological properties of volumetric structure as three-dimensional objects. Although local structure of them as volumetric data are computed from geometric and topological properties, texture allows to estimate both local and global statistical properties of these objects. For the data analysis of these volumetric data, methodologies simultaneously to process geometrical and topological structures and texture properties are required. The multi-way PCA of tensor allows pattern recognition, classification, compression and data retrieval of these volumetric objects in medicine and biology possessing both geometric and statistic properties. Extension of the algorithms to higher order multi-way data analysis, such as spatio-temporal volumetric analysis of moving and deforming objects is straightforward.

Names: Luca Calatroni, Adrian Martín and Federica Sciacchitano

Title of contribution: Learning and optimisation approaches for non-standard noise distributions

Abstract of Contribution:
We propose an optimisation strategy to design a tailor-made approach for the optimal denoising of images corrupted by two non-standard noise distributions. Namely, we will consider the Cauchy noise distribution often encountered in Synthetic Aperture Radar (SAR) images and Rician noise distribution, which has been often considered as the appropriate noise model for MRI medical images. The statistical and physical properties of these noise models strongly differ from more common Gaussian, Laplace or Poisson noise distributions. For instance, random variables distributed according to Cauchy distribution do not have finite variance, while Rician random variables show signal dependence. Furthermore, the variational data fidelities derived from these noise models through MAP estimation are non-convex, so their numerical optimisation is rendered hard.

In this work we want to focus on the optimal design of Total Variation (TV) denoising models based on the use of the constrained optimisation methods. Namely, two aspects are considered.
1. In the expression of the data fitting term of both distributions some statistical parameters quantifying the noise level explicitly appear. In order to write down the corresponding TV-denoising model their estimation is then needed. Often, such estimate is based on data-driven approaches, but has never been studied in a learning and optimisation framework;
2. A crucial question in every regularisation approach is the estimation of the optimal parameter weighting the trust in the data against the effect of the regularisation. Assuming a fixed acquisition setting for training set of images at hand, we formulate a bilevel optimisation approach which computes the optimal regularisation parameter by optimising a suitable quality measure.

Preliminary results confirm the effectiveness of our approach and pose several further questions related to the approximation of such non-standard noise distributions via a combination of classical noise models.

Names: Rasmus Dalgas Kongskov, Yiqiu Dong and Kim Knudsen

Title of contribution: Directional Total Variation
Abstract of Contribution:

Regularization is a widely used technique for, among other things, overcoming the stability issues of several inverse problems. Total variation (TV) regularization has been shown to be a very useful tool when the target of interest is piecewise constant. We introduce a reformulation of directional total variation (DTV), based on the dual formulation of TV. DTV is a useful regularization tool for targets which, besides being piecewise constant, includes directional information. We show that DTV has the same essential mathematical properties as TV. Using automated estimation of the main direction in the target, we demonstrate the improvement of using DTV as a regularizer compared to standard TV. For computed tomography (CT) an estimation of the main direction can easily be computed directly from the data. Numerical simulated experiments are carried out for a practical CT-reconstruction problem using DTV regularization. Finally we demonstrate incorporation of directional information in other regularization methods.

Names: Erik Hanson and Erlend Hodneland

Title of contribution: Estimation of realistic deformation fields in 4D image registration

Abstract of Contribution:

Traditionally, image registration can be seen as the process of establishing a meaningful correspondence between a reference and a target image. The model, or the generic rules, used to align the images will often allow many, or perhaps infinity many different solutions to the alignment problem. Thus, regularization terms are used to guide the alignment towards a reasonable solution. In many applications, particularly within non-parametric image registration, regularizers are known within the field of continuum mechanics. These models allow for non-rigid deformations, but will at the same time ensure smooth and physically realistic deformation fields. The latter is particularly important within registration of temporal image sequences of i.e. soft tissue, where the regularizer may incorporate the actual physics of the imaged object. A widely used regularizing term is the linear elastic constitutive equation, valid for smaller deformations originating from breathing and other shallow movements. However, this term does not incorporate any temporal information. To account for this, we have in our work focused particularly on a poro-elastic model. This is a linear elasticity model incorporating time varying deformations and volume changes due to displacement of a fluid in a porous-medium. The displacement of low-permeable fluid has a dampening effect on the deformation and thereby requires a different weighting between data and regularization terms than for i.e. linear elasticity.

The main contribution of this work is the mathematical formulation of an image registration framework focusing on the inverse modeling of the actual physical deformation of the imaged object. As the regularizers now represent real physical interactions, the data term will take the role as a physical force driving the deformation. We discuss the relation between the image-derived forces in data terms such as SSD and the physical forces driving the dynamics of the image. In this context, the model parameters such as bulk and shear modulus are fixed properties of the imaged object and cannot be tuned to improve the fit in the image sequence. Experiments on synthetic image data show that the classical approach with only image-based force terms (data term), the true deformation field will not be correctly determined. To this end, our proposed framework contains a modified data term where forces derived from image features are projected to predefined spatial regions of the image. This approach relates the data term to the segmentation of the image into tissue types, a link previously proven to achieve good registration results. The framework also opens for estimation of non-homogeneous parameter maps of i.e. bulk modulus.
Examples from 3D+time pelvic MR imaging as well as 2D video of swimming fish are shown. In both cases, a ground truth deformation field will not be accessible, but the overall aim will be to quantify local properties of the living tissue. The proposed framework opens the path to new quantitative applications of traditional image registration methods, but may also contributes to the discussion around robust and verifiable methods for 4D image registration.

Names: Samee Maharjan, Ola Marius Lysaker and Andre V. Gaathaug

Title of contribution: FRONT TRACKING OF DETONATION WAVES BY OPEN ACTIVE CONTOUR MODEL

Abstract of Contribution: This paper presents the application of image processing technique on front tracking of different types of waves that are generated during gas explosions. The tracking will eventually use for calculating wave properties like velocity and pressure along the front. An open active contour method using GVF force as an external force is used to find the front of a wave. All experiments are done with premixed hydrogen and air at ambient temperature and pressure. The recorded high speed schlieren photographs are digitally processed and local velocity and pressure are calculated using the normal shock relations. The calculated results are validated with the measured values from sensors, which shows the designed framework work fairly.

Names: Naftali Zon and Nahum Kiryati

Title of contribution: Unified Functional Framework for Restoration of Image Sequences Degraded by Atmospheric Turbulence

Abstract of Contribution: We propose a unified functional to address the restoration of turbulence-degraded images. This functional quantifies the association between a given image sequence and a candidate latent image restoration. Minimizing the functional using the alternating direction method of multipliers (ADMM) and Moreau proximity mapping leads to a general algorithmic flow. We show that various known algorithms can be derived as special cases of the general approach. Furthermore, we show that building-blocks used in turbulence recovery algorithms, such as optical flow estimation and blind deblurring, are called for by the general model. The main contribution of this work is the establishment of a unified theoretical framework for the restoration of turbulence-degraded images. It leads to novel turbulence recovery algorithms as well as to better understanding of known ones. Along the way we provide convergence analysis and guarantees for a novel feedback-framework for blind deblurring. In addition we present a direct scheme for optical flow estimation with non-smooth lp regularization.
Names: Haixia Liu, Jian-Feng Cai and Yang Wang

Title of contribution: Subspace Clustering on Symmetric Positive Definite Manifolds

Abstract of contribution:
High-dimensional data often lie in low-dimensional subspaces instead of the whole space. In this work, we propose a new subspace clustering methods based on symmetric positive definite manifolds.